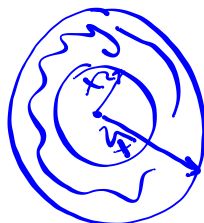


7.2/36

what are the pts of intersection?  
 $x^2 = \sqrt{x} \Rightarrow x^4 = x \Rightarrow x^4 - x = 0 \Rightarrow x(x^3 - 1) = 0$   
 what is the cross-sectional shape?

 $\Rightarrow$  annulus

what is the area  
 of a cross-section?



$$\pi R_0^2 - \pi R_I^2$$

what are the radii?

$$R_0 = \sqrt{x} ; R_I = x^2$$

the area of my cross-section =  $\pi(\sqrt{x})^2 - \pi(x^2)^2$

Volume =  $\int_0^1 \pi(\sqrt{x})^2 - \pi(x^2)^2 dx =$

$$\pi \int_0^1 x - x^4 dx = \pi \left( \frac{x^2}{2} - \frac{x^5}{5} \right) \Big|_0^1$$

$$= \pi \left( \frac{1}{2} - \frac{1}{5} \right) - 0 = \boxed{\frac{3\pi}{10}}$$

7.2/39 base of solid  $\Rightarrow x^2 + y^2 = 1$

a) shape?

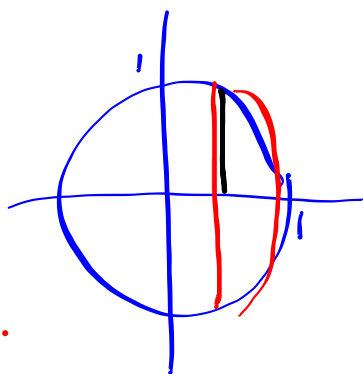
semicircle

area of shape?

$$A = \frac{1}{2} \pi r^2$$

what is  $r$ ?

$$r = y = \sqrt{1 - x^2}$$



$$y^2 = 1 - x^2$$

$$y = \sqrt{1 - x^2}$$

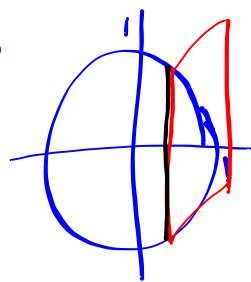
$$A = \frac{1}{2} \pi (\sqrt{1 - x^2})^2$$

$$\text{Volume} = \int_{-1}^1 \frac{\pi}{2} (1 - x^2) dx = \frac{\pi}{2} \int_{-1}^1 (1 - x^2) dx$$

$$= \frac{\pi}{2} \left( x - \frac{x^3}{3} \right) \Big|_{-1}^1 = \frac{\pi}{2} \left[ \left( 1 - \frac{1}{3} \right) - \left( -1 - \frac{-1}{3} \right) \right]$$

$$= \frac{\pi}{2} \left( 2 - \frac{2}{3} \right) = \frac{\pi}{2} \left( \frac{4}{3} \right) = \left( \frac{2\pi}{3} \right)$$

7.2/396



shape?

$\Rightarrow$  square  
area of shape?

$$\Rightarrow A = s^2$$

what is  $s$ ?

$$s = 2y = 2(\sqrt{1-x^2})$$

$$\text{Area of cross section} = s^2 = (2\sqrt{1-x^2})^2$$

$$= 4(1-x^2)$$

$$\text{Volume} = \int_{-1}^1 4(1-x^2) dx = 4\left(x - \frac{x^3}{3}\right) \Big|_{-1}^1$$

$$= 4\left[\left(1 - \frac{1}{3}\right) - \left(-1 - \frac{-1}{3}\right)\right]$$

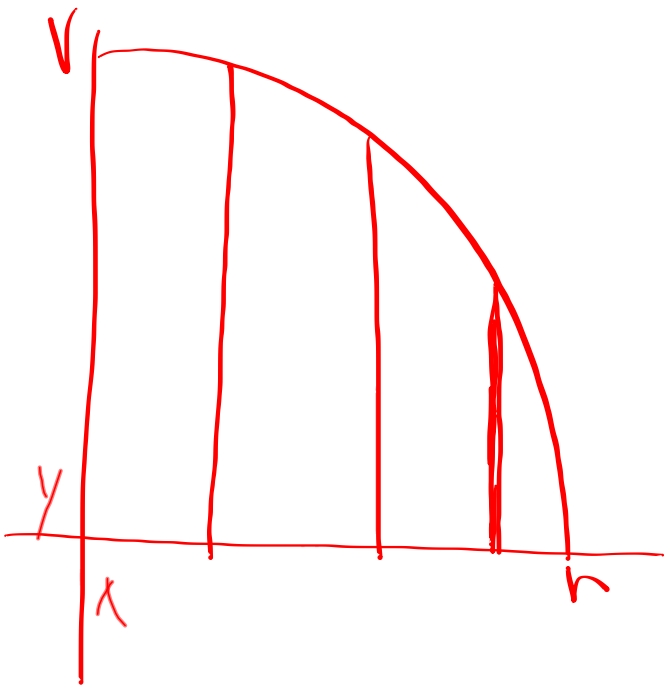
$$= 4\left(\frac{4}{3}\right) = \frac{16}{3}$$

$$V_{\text{ball}} = \frac{4}{3}\pi r^3$$

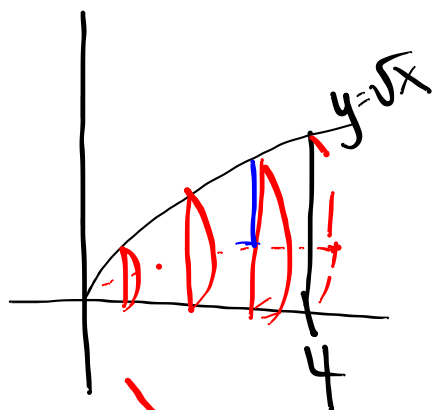
$$SA_{\text{ball}} = 4\pi r^2$$

find  
Vol of cube  
when  
 $SA = V'$

find  
2 or 3  
ways  
to use  
integral  
to get  
Vol of cube



7.2 / 38 base:  $y = \sqrt{x}$ ;  $y = 0$ ;  $x = 4$   
 cross-section: semicircle



area of a  $\Delta$ :  $\frac{\pi r^2}{2}$

what is the  $r$ ?

$$r = \frac{1}{2} y = \frac{1}{2} \sqrt{x}$$

what the limits are:  $x \in [0, 4]$

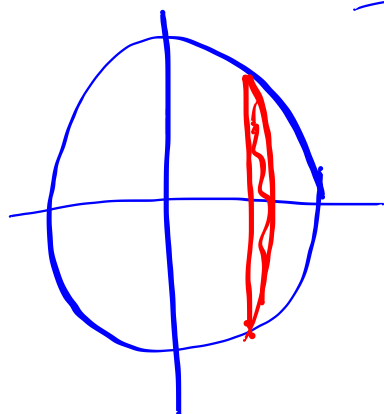
$y = d$

Volume:

$$\int_0^4 \frac{\pi \left( \frac{1}{2} \sqrt{x} \right)^2}{2} dx$$

$$= \frac{\pi}{8} \int_0^4 x dx = \frac{\pi}{8} \left. \frac{x^2}{2} \right|_0^4 = \pi$$

39) ~~part~~ base:  $x^2 + y^2 = 1 \rightarrow y^2 = 1 - x^2$   
 $y = \sqrt{1 - x^2}$   
 cross-section: Semicircle



Area w/  $\Delta = \frac{1}{2} \pi r^2$

what is  $r$ ?

$r = y = \sqrt{1 - x^2}$

limits:  $[-1, 1]$

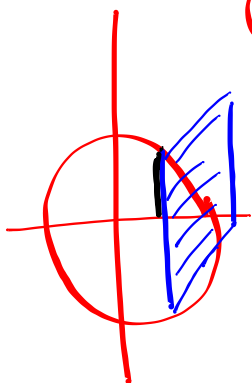
Volume:  $\int_{-1}^1 \frac{\pi}{2} (\sqrt{1 - x^2})^2 dx = \frac{\pi}{2} \int_{-1}^1 (1 - x^2) dx$

$= \frac{\pi}{2} \left( x - \frac{x^3}{3} \right) \Big|_{-1}^1 = \frac{\pi}{2} \left( 1 - \frac{1}{3} - \left( -1 - \frac{-1}{3} \right) \right)$

$= \frac{\pi}{2} \left( 2 - \frac{2}{3} \right) = \frac{\pi}{2} \left( \frac{4}{3} \right) = \frac{2\pi}{3}$

b: base:  $x^2 + y^2 = 1$

Cross-section: Squares

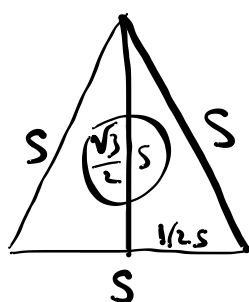


area of  $\square = s^2$   
what is  $s$ ?

$$s = (2y) = 2\sqrt{1-x^2}$$

$$\begin{aligned} \text{Volume} &= \int_{-1}^1 (2y)^2 dx = \int_{-1}^1 4(\sqrt{1-x^2})^2 dx \\ &= 4 \int_{-1}^1 1-x^2 dx = 4 \left( x - \frac{x^3}{3} \right) \bigg|_{-1}^1 = 4 \left( \frac{4}{3} \right) = \frac{16}{3} \end{aligned}$$

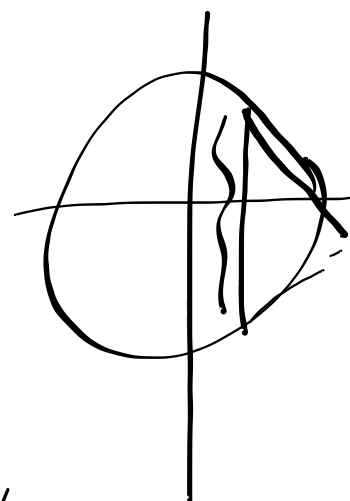
39c)

base:  $x^2 + y^2 = 1$ cross-section: equilateral triangle

$$\text{area} = \frac{\sqrt{3}}{4} s^2$$

what is  $s$ ?

$$s = 2y = 2\sqrt{1-x^2}$$



$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}(s)\left(\frac{\sqrt{3}}{2}s\right)$$

$$V = \int_{-1}^1 \frac{\sqrt{3}}{4} (2\sqrt{1-x^2})^2 dx$$

$$= \sqrt{3} \left( x - \frac{x^3}{3} \right) \Big|_{-1}^1 = \sqrt{3} \left( \frac{4}{3} \right) = \frac{4\sqrt{3}}{3}$$





